

# Phase retrieval from spectral interferograms including a stationary-phase point

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## Abstract

We report on a simple processing procedure to retrieve the phase from spectral interferograms including a stationary-phase point. First, the numerical simulations are performed to demonstrate high precision of the phase retrieval from the spectral interferogram. Second, the feasibility of the procedure is confirmed in processing experimental data from a dispersive Michelson interferometer comprising a cube beam splitter and a plate made of BK7 optical glass. From the retrieved spectral phase, the effective thickness of the BK7 optical glass is determined precisely.

*Key words:* white light, spectral interferometry, dispersion, retrieved phase, cube beam splitter, plate, BK7 optical glass, effective thickness

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## 1 Introduction

White-light spectral interferometry utilizing interference of two beams from a broadband source in a standard Michelson or a Mach-Zehnder interferometer has been widely used in various research areas including distance and displacement measurements [1–5], profilometry [6–9], material characterization [5,10–18] and optical communications [19]. It has also become a very useful tool for a fiber-based sensing [20,21].

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White-light spectral interferometry is based on the observation of spectrally-resolved interference fringes (channeled spectrum, spectral interferogram) and involves measurement of the phase or period of the spectral fringes. The number of the spectral fringes resolved in an operational range of a spectrometer depends on the optical path difference (OPD) adjusted in the interferometer [2], on the width of the response function (the resolving power) of the spectrometer and on the amount of dispersion in the interferometer [4]. For non-dispersive interferometers, the period of the spectral fringes is inversely proportional to the OPD adjusted in the interferometer. In profilometry, for example, the sample height profile can be determined by simply differentiating the spectral phase difference (the spectral phase) retrieved from a single interferogram recorded in the wavenumber domain [6–9].

For dispersive interferometers, the period of the spectral fringes is inversely proportional to the group OPD between beams of the interferometer so that a stationary-phase point [22–25] with the highest visibility of the spectral fringes can be resolved in the recorded spectral interferogram for the zero group OPD. The group dispersion of a sample of the known thickness placed in a non-dispersive interferometer, which shows up as a dispersive interferometer, can be obtained using two approaches. First, by simply differentiating the frequency-dependent spectral phase retrieved from a spectral interferogram recorded far from the stationary-phase point [19]. Second, using the dependence of a wavelength position of the stationary-phase point on the position of the one of the interferometer mirrors [25].

Among the techniques used in processing spectral interferograms not including the stationary-phase point are those that require only one spectral interferogram to retrieve the spectral phase [26]. These include a two-point [12], a five-point [2,10] and a seven-point [9] algorithms, Fourier transform [5,6,8], phase-locked loop [13], spatial phase shifting [7], Hilbert transform [27], wavelet transform [15], windowed Fourier transform [16] and fringe counting [18] techniques.

Among the techniques used in processing the spectral interferograms including the stationary-phase point are those that utilize a fit of the recorded spectral interferogram to the theoretical one [23,24] or are based on processing a series of spectral interferograms [25]. In the former case, the spectral phase is reconstructed from the spectral interference signal [16], which is evaluated from the reference (unmodulated) spectrum obtained by an additional measurement. The main limitation of the method is reached for thick or strongly dispersive materials because under such conditions the spectral interference fringes are resolvable in a narrow spectral range around the stationary-phase point. In the latter case, the group dispersion of a sample of the known thickness placed in the interferometer is measured from the wavelength position of the stationary-phase point (appears in the recorded spectral interferogram when the group

OPD between beams in the interferometer is close to zero) as a function of the position of the one of the interferometer mirrors. Unfortunately, there are also cases when the reference spectrum cannot be obtained by any measurement. These include the spectral interferograms arising due to interference of polarization modes [21] or spatial modes [29] guided by optical fibers.

The aim of the paper is to present a simple processing procedure to retrieve the spectral phase from a single spectral interferogram including a stationary-phase point and resolved over a spectral region as wide as possible (e.g., comparable to the full bandwidth of a white-light source) even if the reference spectrum is not known. First, we performed the numerical simulations to demonstrate high precision of the phase retrieval from the spectral interferogram. Then the feasibility of the method was confirmed in processing experimental data from a dispersive Michelson interferometer to determine the effective thickness of a combination of a cube beam splitter and a plate made of BK7 optical glass. The spectral phase was retrieved and the effective thickness was obtained precisely from the slope of the linear dependence of the retrieved absolute OPD on the known refractive index of BK7 optical glass.

## 2 Theoretical background

### 2.1 Spectral interferogram for a dispersive Michelson interferometer

First, let us consider an ideal (non-dispersive) Michelson interferometer with the OPD  $\Delta_M = 2(L-l)$  adjusted between beams of the interferometer, where  $l$  and  $L$  are the optical path lengths in the air in the first and in the second arm of the interferometer, respectively. Then let us consider that the interferometer is replaced by a real (dispersive) Michelson interferometer including a plate beam splitter (see Fig. 1) of the effective thickness  $t_{\text{eff}}$  and the refractive index  $n(\lambda)$ . The OPD  $\Delta_M(\lambda)$  between beams of the dispersive interferometer is given by

$$\Delta_M(\lambda) = 2(L-l) + 2t_{\text{eff}}[n(\lambda) - 1]. \quad (1)$$

The two-beam interference can be resolved at the output of the interferometer as a spectral interferogram

$$I_M(\lambda) = I_0(\lambda)\{1 + V_I(\lambda) \cos[(2\pi/\lambda)\Delta_M(\lambda)]\}, \quad (2)$$

where  $I_0(\lambda)$  is the reference spectrum and  $V_I(\lambda)$  is the overall visibility, which can be expressed for a spectrometer of a Gaussian response function as [4]

$$V_I(\lambda) = V_I \exp\{-(\pi^2/2)[\Delta_M^g(\lambda)\Delta\lambda_R/\lambda^2]^2\}, \quad (3)$$

where  $V_I$  is a visibility term,  $\Delta\lambda_R$  is the width of the spectrometer response function and  $\Delta_M^g(\lambda)$  denotes the group OPD between beams of the dispersive interferometer given by

$$\Delta_M^g(\lambda) = 2(L - l) + 2t_{\text{eff}}[N(\lambda) - 1], \quad (4)$$

where  $N(\lambda)$  is the group refractive index defined as

$$N(\lambda) = n(\lambda) - \lambda \frac{dn(\lambda)}{d\lambda}. \quad (5)$$

The cosine term in Eq. (2) can be expressed as the normalized spectral signal  $S_M(\lambda)$  defined by

$$S_M(\lambda) = \frac{I(\lambda) - I_0(\lambda)}{I_0(\lambda)V_I(\lambda)} = \cos[(2\pi/\lambda)\Delta_M(\lambda)]. \quad (6)$$

When the case of a thick plate of the beam splitter material (e. g., fused silica or BK7 optical glass) is considered, the spectral interference fringes have the highest visibility and the largest period in the vicinity of a stationary-phase point with the zero group OPD at one specific wavelength  $\lambda_0$ , which is referred to as the equalization wavelength [25] and which satisfies the relation

$$2(L - l) + 2t_{\text{eff}}[N(\lambda_0) - 1] = 0. \quad (7)$$

From Eq. (7) it results that the mirror position  $L = L(\lambda_0)$  with the equalization wavelength  $\lambda_0$  resolved in the recorded spectral interferogram is given by the relation

$$L(\lambda_0) = l - t_{\text{eff}}[N(\lambda_0) - 1]. \quad (8)$$

If we introduce the mirror displacement  $\Delta L(\lambda_0) = L(\lambda_0) - L(\lambda_{0r})$  as the displacement of mirror 2 of the dispersive interferometer from the position with the reference equalization wavelength  $\lambda_{0r}$ , we obtain for the differential group refractive index  $\Delta N(\lambda_0) = N(\lambda_0) - N(\lambda_{0r})$  the relation

$$\Delta N(\lambda_0) = -\Delta L(\lambda_0)/t_{\text{eff}}. \quad (9)$$

According to Eq. (9), the differential group refractive index  $\Delta N(\lambda_0)$  can be measured directly as a function of the equalization wavelength  $\lambda_0$  when the effective thickness  $t_{\text{eff}}$  of a beam splitter is known [25]. Similarly, from Eq. (9) written in the form

$$\Delta L(\lambda_0) = -\Delta N(\lambda_0)t_{\text{eff}} \quad (10)$$

it results that the effective thickness  $t_{\text{eff}}$  of a beam splitter is given by a slope of the linear dependence of the mirror displacement  $\Delta L(\lambda_0)$  on the known differential group refractive index  $\Delta N(\lambda_0)$  [25]. Limitations of this method are a precise adjustment of positions of mirror 2 and resolving a series of spectral interferograms with equalization wavelengths in a spectral range as wide as possible. We propose a much simpler approach of determining the effective thickness, based on the use of a phase retrieval procedure for only one spectral interferogram.

## 2.2 Numerical simulation and phase retrieval

In this subsection we show the feasibility of a simple phase retrieval procedure in processing the simulated spectral interferogram, including the noise, related to a dispersive Michelson interferometer. To simulate the interference signal  $I_M(\lambda)$ , we consider a Michelson interferometer with two identical metallic mirrors and a plate beam splitter of the effective thickness  $t_{\text{eff}} = 5000 \mu\text{m}$  made of BK7 optical glass, the wavelength-dependent refractive index  $n(\lambda)$  of which can be approximated by the semi-empirical Sellmeier expression

$$n^2(\lambda) = 1 + \sum_{i=1}^3 \frac{A_i \lambda^2}{(\lambda^2 - B_i)}, \quad (11)$$

where the coefficients are for the temperature of  $20^\circ\text{C}$  and the wavelength in the micrometers as follows [28]:  $A_1 = 1.03961212$ ,  $A_2 = 0.231792344$ ,  $A_3 = 1.01046945$ ,  $B_1 = 6.00069867 \times 10^{-3}$ ,  $B_2 = 2.00179144 \times 10^{-2}$  and  $B_3 = 103.560653$ . The group refractive index  $N(\lambda)$ , which is given by Eq. (5), can be expressed as

$$N(\lambda) = n(\lambda) + \frac{\lambda^2}{n(\lambda)} \sum_{i=1}^3 \frac{A_i B_i}{(\lambda^2 - B_i)^2}. \quad (12)$$

Then we choose the reference spectrum  $I_0(\lambda)$  (known from measurement and affected by the noise) and calculate the spectral interferogram  $I_M(\lambda)$  for 2048 wavelengths with an equal sampling in the range from 350 to 1000 nm. Fig. 2 shows the reference spectrum by the dashed line and the spectral interferogram

by the solid line in a narrower wavelength range from 500 to 950 nm. The simulated spectral interferogram given by Eq. (2) is calculated for the width of the spectrometer response function  $\Delta\lambda_R=3$  nm, the visibility term  $V_I=0.9$  and such optical path lengths  $l$  and  $L$  adjusted in the arms of the interferometer so that a stationary-phase point at a wavelength of 639.05 nm is resolved.

The method of processing is based on the approach described in [20], designed for retrieving the spectral phase from a single spectral interferogram not including a stationary-phase point. At the first step of the signal processing, the original spectral interferogram has to be examined for the loss of fringe contrast, which decreases the accuracy of detection of the fringe extrema. In such a case the signal has to be resampled with higher sampling density using interpolation. Then the resampled spectral interferogram is smoothed to eliminate false maxima and minima caused by noise. In the method we used a Hamming window-based, zero-phase low-pass finite impulse response filter. After these steps we obtained smoothed spectrum  $I_r(\lambda)$  ready for further processing.

Following the procedure described in [20], we retrieved two auxiliary spectra: upper  $I_{rmax}(\lambda)$  and lower  $I_{rmin}(\lambda)$  envelopes of the spectral interferogram. The envelopes were obtained by interpolating the maxima and minima of the smoothed spectrum  $I_r(\lambda)$ . Since the processed spectral interferogram contains a stationary-phase point, there is an issue of obtaining the correct shape of envelopes, which collapse at this wavelength. In our approach we simply remove the stationary point from a list of extrema and take advantage of smoothing property of cubic spline interpolation. Using the retrieved envelopes we can calculate the normalized spectral interference signal  $S_{Mr}(\lambda)$  or the cosine term [see Eq. (6)] as

$$S_{Mr}(\lambda) = \frac{2I_r(\lambda) - [I_{rmax}(\lambda) + I_{rmin}(\lambda)]}{I_{rmax}(\lambda) - I_{rmin}(\lambda)} = \cos[\Phi_r(\lambda)]. \quad (13)$$

The normalized spectral interference signal retrieved from the simulated spectral interferogram is shown in Fig. 3. The spectral phase  $\Phi_r(\lambda)$  is retrieved and unwrapped based on Eq. (13). One should note that proper unwrapping should start at the stationary phase point and proceed independently towards short and long wavelengths.

The unwrapped phase function  $\Phi_r(\lambda)$  is known with the ambiguity of  $m2\pi$ , where  $m$  is an integer. To obtain the corresponding absolute phase function

$$\phi_r(\lambda) = (2\pi/\lambda)\Delta_{Mr}(\lambda) \quad (14)$$

or the absolute OPD  $\Delta_{Mr}(\lambda)$ , we use a procedure presented in a previous paper [16]. Using Eqs. (1) and (14), the OPD  $\Delta_{Mr}(\lambda)$  between interfering

beams satisfies the relation

$$\Delta_{\text{Mr}}(\lambda) = [\Phi_{\text{r}}(\lambda)/(2\pi) + m]\lambda = 2(L - l) + 2t_{\text{eff}}[n(\lambda) - 1]. \quad (15)$$

It results from Eq. (15) that the knowledge of the unwrapped phase function  $\Phi_{\text{r}}(\lambda)$  and the dispersion of the refractive index  $n(\lambda)$  enables one to determine the interference order  $m$  and thus the absolute spectral phase  $\phi_{\text{r}}(\lambda)$  and OPD  $\Delta_{\text{Mr}}(\lambda)$  [16]. The interference order  $m$  of such a value needs to be chosen so that the OPD  $\Delta_{\text{Mr}}(\lambda)$  between beams in the interferometer is linearly dependent on the refractive index  $n(\lambda)$ . A slope of the corresponding linear function gives the twofold effective thickness  $2t_{\text{eff}}$ .

Using the method we retrieved the phase function  $\phi_{\text{r}}(\lambda)$  in the range from 560 to 930 nm as shown in Fig. 4 together with the modeled phase function, including the stationary-phase point. The precision of the phase retrieval by the method is demonstrated by the phase error distribution which is depicted in Fig. 5 and which does not exceed 0.2 rad. In this figure we clearly see the phase retrieval artifacts around the stationary-phase point. Fig. 6 shows by the solid line the retrieved absolute OPD  $\Delta_{\text{Mr}}(\lambda)$  as a function of the refractive index  $n(\lambda)$  of BK7 optical glass given by Eq. (11) together with the corresponding linear fit that gives the beam splitter effective thickness  $t_{\text{ef}} = 5000.11 \mu\text{m}$  with a standard deviation of  $0.01 \mu\text{m}$ . This value is in an excellent agreement with that used in the model. We can conclude that if the dispersion of the beam splitter material is known, the effective thickness  $t_{\text{eff}}$  can be determined precisely by our method of phase retrieval from one spectral interferogram including a stationary-phase point.

### 3 Experimental configuration

The experimental setup used in the application of spectral-domain white-light interferometry to record spectral interferograms and to process them by the method is shown in Fig. 1. From the retrieved spectral phase, the effective thickness of optical glass of the known dispersion is determined. The setup consists of a white-light source (a halogen lamp with launching optics), optical fiber with a collimating lens, a bulk-optic Michelson interferometer with two metallic mirrors and a cube beam splitter made of BK7 optical glass [16], a micropositioner connected to mirror 2, a microscope objective, micropositioners, a miniature fiber-optic spectrometer S2000 (Ocean Optics) with a read optical fiber, an A/D converter and a personal computer.

A plate made of BK7 optical glass (WG11050, Thorlabs) with a thickness of about 5 mm is placed in the first arm of the interferometer in such a way that

the collimated beam is incident on the surfaces of the plate perpendicularly (the adjustment is performed by a laser diode used instead of the halogen lamp). The resolution of the fiber-optic spectrometer S2000 is in our case given by the effective width of the light beam from a core of the read optical fiber. To assure a Gaussian response function of the spectrometer, we used the read optical fiber of a 50 mm core diameter [4].

## 4 Experimental results and discussion

The overall effective thickness  $t_{\text{eff}}$  of a combination of the cube beam splitter and the plate (both made of BK7 optical glass) was measured by the method based on phase retrieval from one spectral interferogram including a stationary-phase point. Example of the spectral interferogram recorded for a suitable OPD adjusted in the Michelson interferometer is shown in Fig. 7. In the first step of the procedure the reference and envelope spectra were determined and they were used to retrieve the normalized spectral interference signal  $S_{\text{Mr}}(\lambda)$  given by Eq. (13) and shown in Fig. 8. The retrieved spectral interference signal has a stationary phase point at a wavelength of 655.42 nm.

By processing the spectral interference signal using an unwrapping algorithm the unwrapped phase function  $\Phi_{\text{r}}(\lambda)$  was determined. From the function  $\Phi_{\text{r}}(\lambda)$  the absolute phase function  $\phi_{\text{r}}(\lambda)$  was determined by a procedure presented above. Such an interference order  $m$  was used so that the OPD  $\Delta_{\text{Mr}}(\lambda)$  given by Eq. (15) is linearly dependent on the refractive index  $n(\lambda)$  of BK7 optical glass. This is illustrated in Fig. 9 by the solid line together with the corresponding linear fit that gives the overall effective thickness  $t_{\text{eff}} = 5160.8 \mu\text{m}$  with a standard deviation of  $0.1 \mu\text{m}$ . The negative slope of this linear dependence means that the plate is placed in the other arm of the interferometer than it is considered in the numerical simulation (see Fig. 1). The overall effective thickness obtained can be compared with that determined from the measured dependence of the precise position of mirror 2 of the interferometer on the corresponding equalization wavelength as demonstrated in a previous paper [25]. This procedure gives the overall effective thickness  $t_{\text{eff}} = 5155 \mu\text{m}$  with a standard deviation of  $6 \mu\text{m}$ , which is in good agreement with the value obtained by the method based on the phase retrieval from a single spectral interferogram.

## 5 Conclusions

A simple method of processing the spectral interferograms including a stationary-phase point has been presented. The numerical simulations have been per-



formed to demonstrate high precision of the phase retrieval from the spectral interferogram. Phase error distribution has also been specified. The feasibility of the method, which is easy to implement, has been confirmed in processing experimental data from a dispersive Michelson interferometer to determine precisely the effective thickness of a combination of a cube beam splitter and a plate made of BK7 optical glass. The spectral phase was retrieved to obtain the effective thickness from the slope of the linear dependence of the retrieved OPD on the refractive index of the glass. The effective thickness obtained by this procedure was compared with that obtained by a different procedure, and good agreement was confirmed.

The results obtained are important from the point of view of processing the spectral interferograms including a stationary-phase point. These can be the results of implementation of white-light spectral interferometry in various research fields, including distance and displacement measurements, profilometry, material characterization, optical communications and sensing. The phase retrieval procedure is especially important for the spectral interferograms arising due to interference of polarization modes [21] or spatial modes [29] guided by optical fibers when the reference spectrum cannot be measured.

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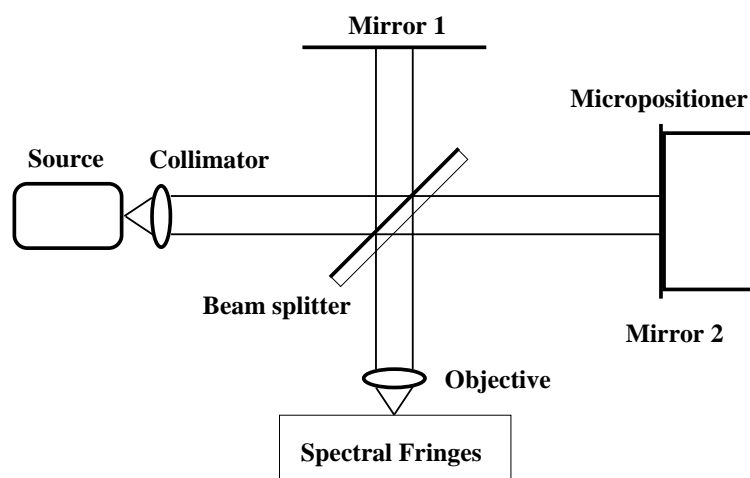


Fig. 1. Experimental setup with a dispersive Michelson interferometer to record spectral interferograms.

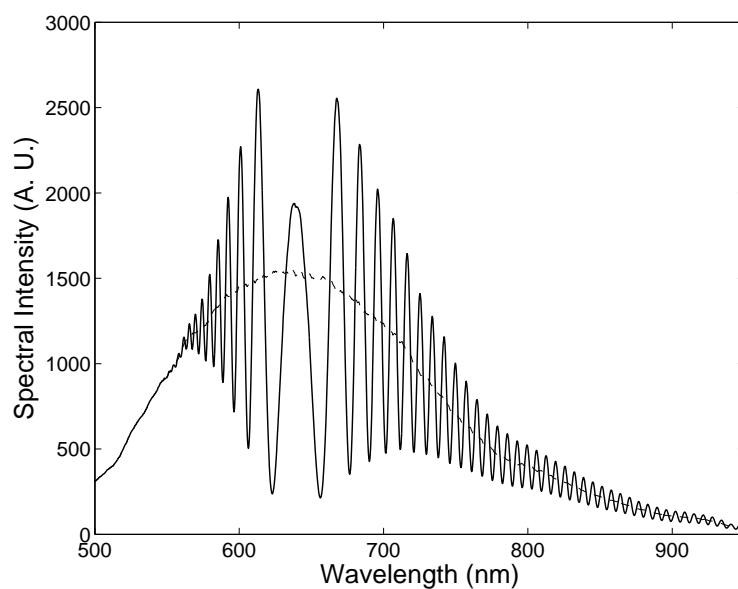


Fig. 2. Theoretical spectral interferogram (solid) and the reference spectrum (dashed).

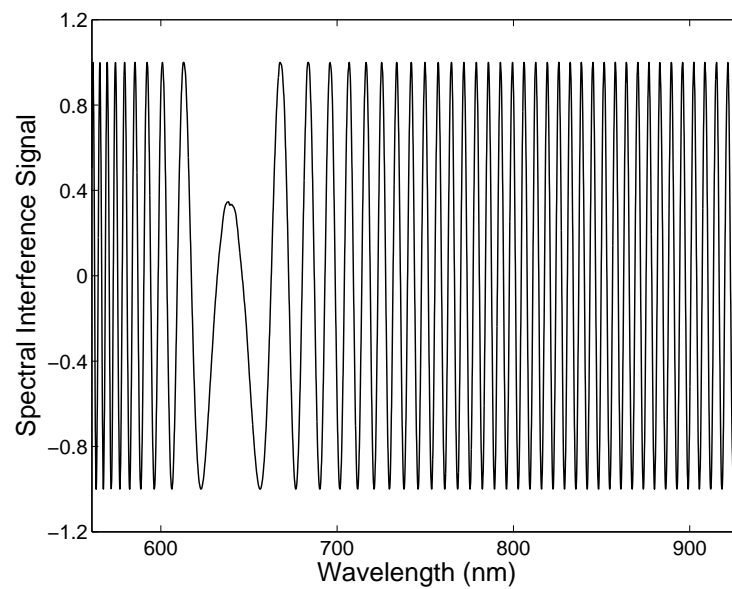


Fig. 3. Normalized spectral interference signal retrieved from the theoretical spectral interferogram shown in Fig. 2

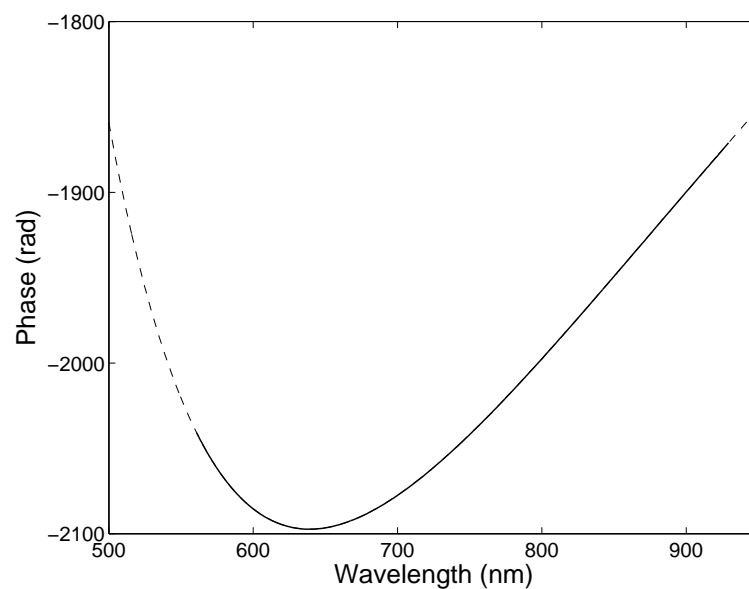


Fig. 4. Comparison of the retrieved phase (solid) with the theoretical one (dashed).

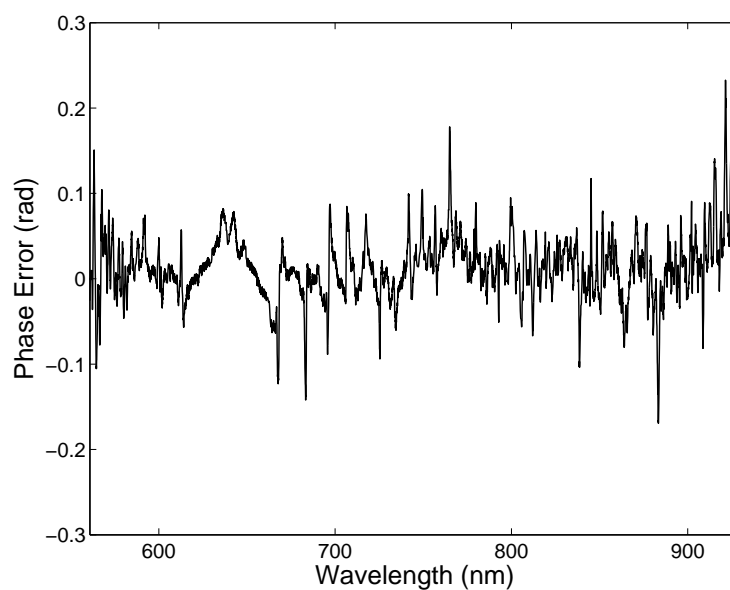


Fig. 5. Error distribution of the retrieved phase.

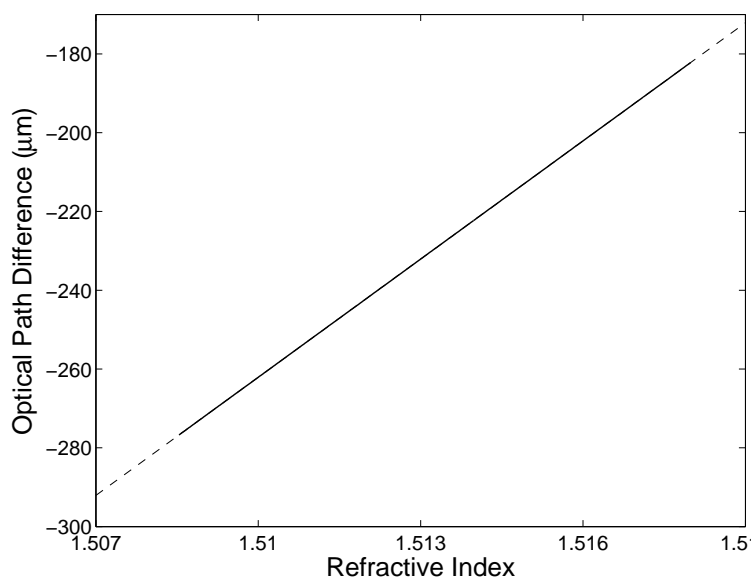


Fig. 6. The OPD between interfering beams as a function of the refractive index of BK7 optical glass (solid) together with a linear fit (dashed).

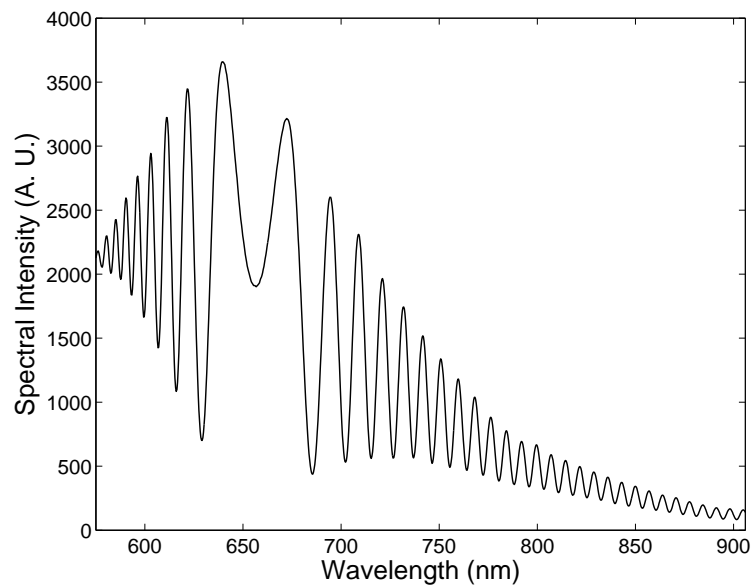


Fig. 7. Example of the recorded spectral interferogram.

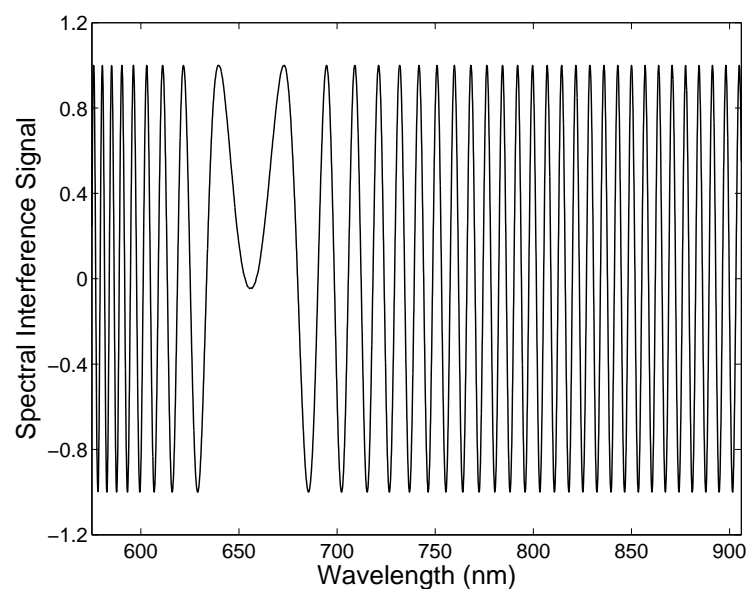


Fig. 8. Normalized spectral interference signal retrieved from the spectral interferogram shown in Fig. 7

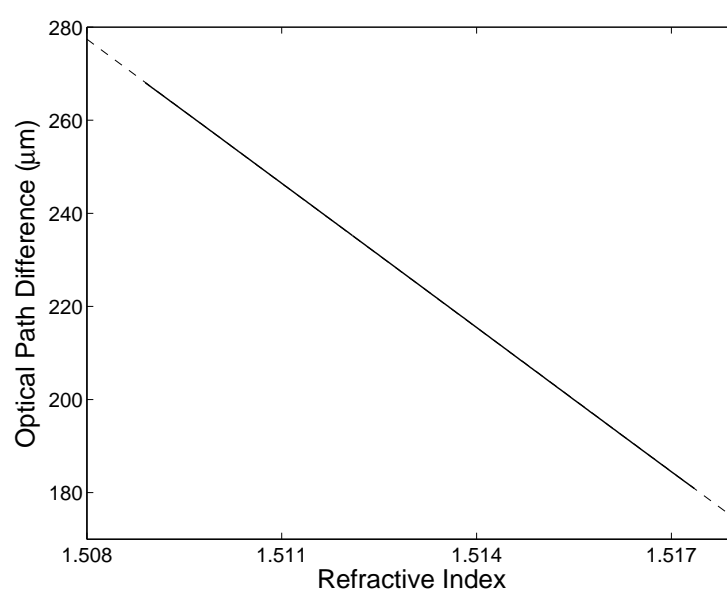


Fig. 9. The OPD between interfering beams as a function of the refractive index of BK7 optical glass (solid) together with a linear fit (dashed).